EXPLORING THE UNCERTAINTIES ON THE FITTING PARAMETERS OF A WIDELY USED MATHEMATICAL MODEL OF X RAY TRANSMISSION

P.R. COSTA, D.V. VIEIRA

Departamento de Física Nuclear – Instituto de Física
Universidade de São Paulo,
São Paulo – SP, Brazil.

E-mail address of main author: pcosta@if.usp.br

Abstract

A group of simulated X ray spectra was used for the calculation of aluminum transmission curves generated considering different applied voltages and additional filtrations. These spectra were modelled by the method introduced by Tucker *et al.* (Med. Phys. 18, 1991) and modified by Costa *et al.* (Health Phys. 92, 2007). The transmission curves were fitted by Archer Equation (Med. Phys. 21, 1994) by using a non-linear least square method using the Origin 8 (Origin Lab, inc.) software. The selected voltage-filtration combinations comply with the RQR requirements adopted by IAEA/TRS457 document. Eighteen transmission data point were simulated for each RQR set considering crescent number of data, from 3 to 200 points by curve. After fitting the curves, the uncertainties on the α, β and γ parameters were obtained and evaluated according to the considered number of points used. The uncertainties in the parameters were reduced approximately by a factor $N^{1/2}$, with N representing the number of transmission points used for generating the fitting parameters□□ in the considered data point range. The presented result allows planning measuring experiments for the implementation of RQR beam qualities according to the expected level of uncertainties required by the Laboratory (PSDL, SSDL or other). Therefore, the time-consuming measuring procedure for establishing radiation qualities can be adequately optimized.

1. INTRODUCTION

The attenuation of an X ray spectrum by a given material of a thickness *x* can be described by the following equation:

$$N(E,x) = N(E,0)e^{-\mu(E)x} \qquad (1)$$

In this equation, $N(E, x)$ is the distribution of photons after the primary spectrum $N(E,0)$ interacts with the attenuating material with known linear attenuation coefficient □(E). This equation is valid in narrow beam conditions, when lower energy photons scattered in the attenuation material do not affect significantly the resulting harder transmitted spectrum.

This equation can be used for simulating a transmission curve by the integration in the energy variable:

$$K(x) = \int_0^{E_{\max}} N(E,x)dE = \int_0^{E_{\max}} N(E,0)e^{-\mu(E)x}dE \qquad (2)$$

In the present work the functions $N(E,0)$ were simulated by using the modified semi-empirical TBC model[1]. The selected voltage-filtration combinations chosen for the simulations complied with the RQR requirements adopted by IAEA/TRS457 document [2].

Each simulated transmission curve was fitted by the parametric equation proposed by Archer et al [3].

$$K(x) = K_0\left[\left(1+\frac{\beta}{\alpha}\right)e^{\alpha\gamma x} - \frac{\beta}{\alpha}\right]^{-1/\gamma} \qquad (3)$$

In this equation, $K(x)$ represents the air-kerma transmitted by a thickness x of a given material, K_0 is non-attenuated air-kerma, and α, β and γ are fitting parameters. These parameters were obtained by applying a non-linear least square method to empirical or simulated transmission data.

2. METHODOLOGY

Table 1 shows the parameters used for simulating the RQR spectra and the corresponding control parameters. These spectra were simulated by using the RQR filters which give the better fit between the TRS457 document reference data for the first HVL, HVL_1, and homogeneity coefficient, h, and the corresponding calculated results for these parameters. The resulting transmission data for this case were fitted according to Archer method and the corresponding uncertainties on the fitting parameters were used for evaluating the behavior of these uncertainties with the number of transmission points.

TABLE I. PARAMETERS USED FOR SIMULATING THE RQR SPECTRA AND THE CORRESPONDING CONTROL PARAMETERS.

Beam code	Voltage (kV)	RQR filter (mm Al)	Number of transmission points	Control parameters			
				TRS 457		Simulation	
				HVL_1 (mmAl)	h	HVL_1 (mmAl)	h
RQR2	40	3.00	3-200	1.42	0.81	1.42	0.82
RQR3	50	3.02	3-200	1.78	0.76	1.78	0.77
RQR4	60	3.30	3-200	2.19	0.74	2.19	0.74
RQR5	70	3.52	3-200	2.58	0.71	2.58	0.72
RQR6	80	3.86	3-200	3.01	0.69	3.01	0.71
RQR7	90	4.24	3-200	3.48	0.68	3.48	0.70
RQR8	100	4.65	3-200	3.97	0.68	4.05	0.71
RQR9	120	5.53	3-200	5.00	0.68	5.00	0.71
RQR10	150	6.85	3-200	6.57	0.72	6.57	0.73

These spectra were simulated by using the RQR filters which give the better fit between the TRS457 document reference data for the first HVL, HVL_1, and homogeneity coefficient, h, and the corresponding calculated results for these parameters.

The uncertainties on the fitting parameters α, β and γ provided by the Levenberg-Marquardt non-linear least square method were calculated by using an algorithm incorporated into the Origin 8.5 software (Origin lab, Inc.). These calculations were performed for each RQR value and each transmission curve obtained with different number of transmission points.

3. RESULTS

Figure 1 shows the plots of the uncertainties on the fitting parameters α, β and γ, respectively □□, □□ and □□□□as a function of the number of points used by the Levenberg-Maquardt fitting algorithm. The data refers to the RQR10 beam (Table 1). The solid curves are fittings for these uncertainties considering the behavior of these uncertainties following the model $\sigma = a \,.\, N^{b}$. Table 2 shows the values of the parameters a and b for the uncertainties calculated according to the RQR10 beam.

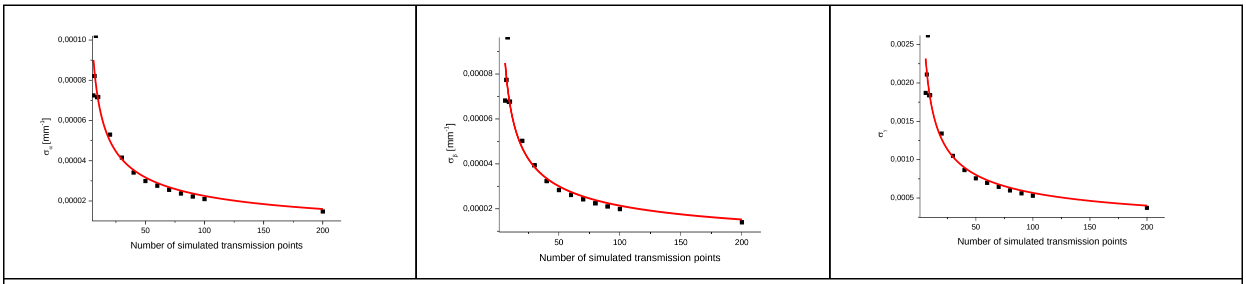

FIG. 1. Uncertainties on the fitting parameters α, β and γ, respectively σ_α, σ_β and σ_γ as a function of the number of points used by the Levenberg-Maquardt fitting algorithm

TABLE II. FITTING PARAMETERS REPRESENTING THE BEHAVIOR OF THE UNCERTAINTIES ON THE PARAMETERS α, β AND γ AS A FUNCTION OF THE NUMBER OF THE TRANSMISSION POINTS, N, FOLLOWING THE MODEl $\boldsymbol{\sigma = a\,.N^{b}}$.

	a	b
σ_α	2.18×10^{-4}	-0.49
σ_β	2.04×10^{-4}	-0.49
σ_γ	5.68×10^{-3}	-0.50

4. CONCLUSION

According to the results presented in Table 2 the uncertainties in the parameters are reduced approximately by a factor $N^{1/2}$, with N representing the number of transmission points used for generating the fitting parameters according to equation (3). The presented result allows planning measuring experiments for the implementation of RQR beam qualities according to the expected level of uncertainties required by the Laboratory (PSDL, SSDL or other). Therefore, the time-consuming measuring procedure for establishing radiation qualities can be adequately optimized.

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